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Supergravity and matrix theory do not disagree on multi-graviton scattering

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Abstract: We compare the amplitudes for the long-distance scattering of three gravitons in eleven dimensional supergravity and matrix theory at finite N. We show that the leading supergravity term arises from loop contributions to the matrix theory effective action that are not required to vanish by supersymmetry. We evaluate in detail one type of diagram—the setting sun with only massive propagators—reproducing the supergravity behavior.

Keywords: M-Theory, D-branes, M(atrix) Theories.

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1. Introduction

In the original work of BFSS [1], it was conjectured that M theory could be described by a matrix model in the large N limit; later, refs. [2, 3, 4] gave meaning to the matrix model also at finite N as describing M theory on the discrete light-cone. Even though, in the long-distance regime, M theory is supergravity in eleven dimensions, there is no guarantee that the supergravity result will always match matrix theory at finite N due to the presence of a small scale in the problem [5]. For this reason, it is of importance to test in actual computations how far the two agree.

The most convincing test to date has been the comparison of the two-body scattering [6, 7].[1] An even more stringent test would come from multi-particle scattering. The case of three 11-dimensional gravitons, carrying Kaluza-Klein momentum in the 10^{th} compactified direction, has been considered in [11] where the authors claim that a term present in the supergravity amplitude cannot arise in the matrix model.

In this note we discuss this important issue by reconsidering the computation of [11]. Our result is encouraging for matrix theory: contrarily to what reported, we find that there are (0+1)-dimensional Yang-Mills (YM) graphs which lead to the same behavior with respect to the relative distances and relative velocities as in the supergravity result.

The diagrams we have considered are two-loop diagrams in the bosonic sector—there are various similar diagrams which can give rise to the same behavior—and we have analyzed in detail one of them, the *setting-sun* diagram with all massive propagators, which only arises in the three-body problem.

When considered in the framework of the effective action arising at one loop by integrating out the long-distance degrees of freedom (the *heavy* modes), our result

[1]For earlier works on different aspects of this problem, see [8, 9, 10].

originates from a term that does not vanish after summing over all bosonic and fermionic contributions—even though the effective operator for the remaining *light* modes does not contain any explicit dependence on the velocity. In the final two-loop effective action, obtained after integrating out all modes, the velocity independent terms cancel (in agreement with supersymmetry [12, 13]) while, the relevant contribution arises from a term of order v^6 which is not expected to vanish by supersymmetry.

We have not attempted to compute the numerical coefficient, which would require the algebraic sum of the various bosonic, fermionic and ghost diagrams of the YM theory at order v^6. Therefore we present here a *minimal* result, which we feel nonetheless to be important because of the recent discussion concerning whether one could or could not find the supergravity behavior in the diagrams of the YM formulation of matrix theory at finite N.

2. The amplitude in supergravity

The simplest way to obtain supergravity amplitudes is by means of string theory. Since it is a tree-level amplitude, it is consistent with conformal invariance in any dimensionality, in particular in $D = 11$. We consider the *bona fide* superstring theory (where there is no tachyon) and the scattering amplitude of three (11-dimensional) gravitons, and look at suitable *pinching* limits, where only intermediate massless states are coupled to the external gravitons. Those states are themselves 11-dimensional gravitons. We then compactify the 10^{th} space dimension giving mass to the external gravitons, which will thus correspond to 10-dimensional $D0$-branes. Keeping zero momentum transfer in the 10^{th} dimension, the intermediate states remain massless and correspond to the various massless fields of 10-dimensional supergravity.

The supergravity amplitude is thus obtained from that of superstring theory by a limiting procedure that isolates the relevant corners of the moduli space. We follow [14], where the appropriate technology is explicitly developed.

By considering only the part of the complete amplitude that is proportional to

$$\varepsilon_1 \cdot \varepsilon_1' \, \varepsilon_2 \cdot \varepsilon_2' \, \varepsilon_3 \cdot \varepsilon_3' \,, \tag{2.1}$$

ε being the external graviton polarization tensor, we obtain the amplitude A_6 for six graviton vertices:

$$\begin{aligned} A_6 = {} & \varepsilon_1 \cdot \varepsilon_1' \, \varepsilon_2 \cdot \varepsilon_2' \, \varepsilon_3 \cdot \varepsilon_3' \, \frac{\kappa^4 (\alpha')^3}{4\pi^3} \int \mathrm{d}^2 x \, \mathrm{d}^2 y \, \mathrm{d}^2 z \, |1-y|^{-2+\alpha' p_2' \cdot p_2} \\ & \times |y|^{\alpha' p_3 \cdot p_2'} |1-x|^{\alpha' p_2 \cdot p_1'} |x|^{\alpha' p_3 \cdot p_1'} |1-z|^{\alpha' p_3' \cdot p_2} \\ & \times |z|^{-2+\alpha' p_3 \cdot p_3'} |z-x|^{\alpha' p_3' \cdot p_1'} |z-y|^{\alpha' p_3' \cdot p_2'} |x-y|^{\alpha' p_2' \cdot p_1'} \\ & \times \left\{ \frac{p_3' \cdot p_1' \, p_2' \cdot p_1'}{(y-x)(z-x)} + \frac{p_3 \cdot p_2' \, p_3' \cdot p_1'}{y(z-x)} - \frac{p_3' \cdot p_2' \, p_3 \cdot p_1'}{x(z-y)} \right. \\ & \left. + \frac{p_2' \cdot p_3' \, p_2' \cdot p_1'}{(y-x)(z-y)} + \frac{p_3' \cdot p_2 \, p_2' \cdot p_1'}{(z-1)(y-z)} \right\} \wedge \left\{ c.c. \right\} . \end{aligned} \tag{2.2}$$

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The eleven-dimensional momenta are chosen to be

$$p_i = (E_i, \mathbf{p}_i - \mathbf{q}_i/2, M_i)\,, \qquad p'_i = (-E'_i, -\mathbf{p}_i - \mathbf{q}_i/2, -M_i)\,, \tag{2.3}$$

where $p_i^2 = 0$, $E_i \simeq M_i + (\mathbf{p}_i - \mathbf{q}_i/2)^2/2M_i$ and $M_i = N_i/R_{11}$ are the momenta in the compactified dimension. Energy-momentum conservation gives $\sum_i \mathbf{q}_i = 0$ and $\sum_i \mathbf{p}_i \cdot \mathbf{q}_i = 0$.

In order to obtain a non-vanishing result in the field theory limit ($\alpha' \to 0$), we must extract three poles in momenta, each of them bringing down one power of $(\alpha')^{-1}$ and thus compensating for the three powers of α' in front of A_6; each pole originates from a pinching limit in which some of the Koba-Nielsen variables come close to each other. The pinching limits corresponding to the grouping of the six external vertices into three pairs i, i', with $i = 1$, 2 and 3, give field theory diagrams where the incoming (i) and outgoing (i') particles of each pair describe the world-line of the $D0$-brane number i.

In particular, we are interested in seven pinching limits. One of them corresponds to the so-called Y diagram where each of the three world-lines are coupled to one intermediate massless state, and the three intermediate states meet at a point. We call the corresponding amplitude A_Y. In addition, there are the diagrams where one world-line is coupled to two intermediate states, each of them attached to one of the two other world-lines (with six possible choices). We disregard terms, which are interpreted as *re-scattering* effects, where a world-line interacts successively with two intermediate states, with an external particle's propagator in between. Thus, besides the Y diagram, we are left with diagrams where two intermediate states originate from the same point of one of the world-lines. We denote the corresponding amplitude by $A_\vee$ and, therefore, we have that $A_6 = A_Y + A_\vee$. We keep only those terms giving the maximal singularity in the momentum transfers.

Let us first consider the amplitude (taking, for the moment, $N_i = 1$)

$$\begin{aligned} A_\vee = {} & 2\,\kappa^4\,\varepsilon_1 \cdot \varepsilon'_1\,\varepsilon_2 \cdot \varepsilon'_2\,\varepsilon_3 \cdot \varepsilon'_3 \\ & \times \left\{ \frac{(\mathbf{p}_3 - \mathbf{p}_2)^2\,(\mathbf{p}_3 - \mathbf{p}_1)^2\,(\mathbf{p}_2 - \mathbf{p}_1)^2}{\mathbf{q}_1^2\,\mathbf{q}_2^2\,\mathbf{q}_3^2} \left(\frac{\mathbf{q}_1^2 + \mathbf{q}_2^2 + \mathbf{q}_3^2}{2} \right) \right\} . \end{aligned} \tag{2.4}$$

Equation (2.4) is the same supergravity amplitude considered in [11].

Equation (2.4) contains three possible singular configurations in which two of the $\mathbf{q}_i$'s are small, describing by Fourier transform the large-distance interaction of one $D0$-brane with the other two. Consider the case where the distance of the brane number 1 from the branes number 2 or 3 is much larger than the distance between the branes 2 and 3. The two singular terms proportional to

$$\frac{1}{\mathbf{q}_1^2 \mathbf{q}_2^2} \quad \text{or} \quad \frac{1}{\mathbf{q}_1^2 \mathbf{q}_3^2} \tag{2.5}$$

are relevant. Let us consider the first one, as the second one is obtained by interchanging 2 with 3.

The Fourier transform gives ($\mathbf{r}_i$ being the position in space of the i-th brane):

$$a_\vee = 2\,\kappa^4\,\varepsilon_1\cdot\varepsilon_1'\,\varepsilon_2\cdot\varepsilon_2'\,\varepsilon_3\cdot\varepsilon_3'\;(\mathbf{p}_3-\mathbf{p}_2)^2\,(\mathbf{p}_3-\mathbf{p}_1)^2\,(\mathbf{p}_2-\mathbf{p}_1)^2\times$$
$$\times\int\frac{\mathrm{d}^9\mathbf{q}_1\mathrm{d}^9\mathbf{q}_2}{(2\pi)^{18}}\frac{1}{\mathbf{q}_1^2\,\mathbf{q}_2^2}\;\exp\Big[i\,\mathbf{q}_1\cdot(\mathbf{r}_1-\mathbf{r}_3)+i\,\mathbf{q}_2\cdot(\mathbf{r}_2-\mathbf{r}_3)\Big]\,. \tag{2.6}$$

To get the case of generic N_i, we have to replace

$$(\mathbf{p}_i-\mathbf{p}_j)^2\to\frac{M_j}{M_i}\mathbf{p}_i^2+\frac{M_i}{M_j}\mathbf{p}_j^2-2\mathbf{p}_i\cdot\mathbf{p}_j \tag{2.7}$$

and write the momenta in terms of the velocities as $\mathbf{p}_i = M_i\mathbf{v}_i$ while bearing in mind that $M_i\sim N_i$. We normalize the amplitude by dividing the result by the product of the M_i and obtain:

$$a_\vee\sim\frac{N_1N_2N_3(\mathbf{v}_3-\mathbf{v}_2)^2(\mathbf{v}_3-\mathbf{v}_1)^2(\mathbf{v}_2-\mathbf{v}_1)^2}{|\mathbf{r}_1-\mathbf{r}_2|^7|\mathbf{r}_2-\mathbf{r}_3|^7}\,. \tag{2.8}$$

In order to compare (2.8) with matrix theory we consider the *eikonal expression* where we integrate over the time t along the world-line trajectories. For simplicity we take the velocities of all three particles to be along the X^1 axis and the relative displacements to be purely transverse (impact parameters). In other words, for the i-th particle, $\mathbf{r}_i = (v_i\hat{\mathbf{n}}_1 t+\mathbf{b}_i)$, with $\mathbf{b}_i\cdot\hat{\mathbf{n}}_1 = 0$. This integral gives, in the limit where $B\equiv|\mathbf{b}_1-\mathbf{b}_2|\gg b\equiv|\mathbf{b}_2-\mathbf{b}_3|$,

$$\tilde a_\vee\sim\int\mathrm{d}t\;\frac{N_1N_2N_3v_{23}^2v_{13}^2v_{12}^2}{(v_{23}^2t^2+B^2)^{7/2}(v_{12}^2t^2+b^2)^{7/2}}\sim\frac{N_1N_2N_3|v_{23}|v_{13}^2v_{12}^2}{B^7b^6}\,, \tag{2.9}$$

where $v_{ij}\equiv v_i-v_j$. The other term in (2.5) gives the same amplitude with B replaced by $B'\equiv|\mathbf{b}_1-\mathbf{b}_3|\simeq B$. It is the amplitude $\tilde a_\vee$ in (2.9) that we want to reproduce in the matrix theory computation.

As for the Y diagram, the corresponding eikonal expression $\tilde a_Y$ turns out to be sub-leading in our limit (see the appendix A).

3. The amplitude in matrix theory

The derivation of the Feynman rules and the computation of the relevant diagrams follow closely those of [6]. We use units where

$$g_{\rm YM}=\left(R_{11}/\lambda_{\rm P}^2\right)^{3/2}=1\,, \tag{3.1}$$

the quantities R_{11}, $\lambda_{\rm P}$ and $g_{\rm YM}$ being the compactification radius, the Planck length and the Yang-Mills coupling, respectively. The bosonic part of the gauge fixed action reads [6]

$$S=\int\mathrm{d}t\;\;\mathrm{Tr}\,\Big(\dot a_0^2+\dot x_i^2+4\,i\,\dot R_k\,[a_0,x_k]-[R_k,a_0]^2-[R_k,x_j]^2$$
$$+\,2\,i\,\dot x_k\,[a_0,x_k]+2\,[R_k,a_0][a_0,x_k]-2\,[R_k,x_j][x_k,x_j]$$
$$-\,[a_0,x_k]^2-\frac{1}{2}[x_k,x_j]^2\Big)\,, \tag{3.2}$$

where a_0 and x_k are hermitian matrices representing the fluctuations and R_k is the background. Since we are studying the scattering of three $D0$-branes, with two independent velocities and impact parameters, we need to consider at least a rank two group, namely $SU(3)$. The overall factors of N_i, representing the longitudinal momentum will be fixed at the end.

We choose the same background that led to (2.9), namely,

$$R_1 = \begin{pmatrix} v_1 t & 0 & 0 \\ 0 & v_2 t & 0 \\ 0 & 0 & v_3 t \end{pmatrix} \qquad \text{and} \qquad R_k = \begin{pmatrix} b_k^1 & 0 & 0 \\ 0 & b_k^2 & 0 \\ 0 & 0 & b_k^3 \end{pmatrix} \quad k > 1\,. \tag{3.3}$$

We factor out the motion of the center of mass by imposing $v_1 + v_2 + v_3 = 0$ and $\mathbf{b}^1 + \mathbf{b}^2 + \mathbf{b}^3 = 0$.

We use a Cartan basis for $SU(3)$, where H^1 and H^2 denote the generators of the Cartan sub-algebra and E_α ($\alpha = \pm\alpha^1, \pm\alpha^2, \pm\alpha^3$) the roots. We also define the space vectors

$$\mathbf{R}^\alpha = \sum_{a=1,2} \alpha^a \,\mathrm{Tr}\, \Big(H^a \mathbf{R}\Big)\,. \tag{3.4}$$

With the standard choice of H^a and α, this definition singles out the relative velocities and impact parameters, e.g. $R_1^{\alpha^1} = (v_2 - v_3)t \equiv v^{23} t$ and, for $k > 1$, $R_k^{\alpha^1} = b_k^2 - b_k^3 \equiv b_k^{23}$ together with cyclic permutations. According to the previous section we choose the relative distance of the first particle with the other two to be much larger than the relative distance of particle two and three, in other words, we set

$$|\mathbf{b}^{\alpha^2}| \approx |\mathbf{b}^{\alpha^3}| \approx B \gg |\mathbf{b}^{\alpha^1}| \approx b \quad \text{and} \quad B\, b \gg v\,, \tag{3.5}$$

where in our units v has the same dimensions as b^2.

The propagators and vertices can be easily worked out from the gauge fixed action (3.2), with two points worth stressing: first, the quadratic part (yielding the propagators) is diagonal in root space; second, contrary to the $SU(2)$ case, there are now vertices with three massive particles (corresponding to the three different roots). The second point is particularly crucial because it is from a diagram containing those vertices that we find the supergravity term.

We find twenty real massless bosons and thirty massive complex bosons. We only need consider some of the latter to construct the diagram. Writing $x_k = x_k^a H^a + x_k^\alpha E_\alpha$, with $x_k^{-\alpha} = x_k^{\alpha *}$, we define the propagators as

$$\langle x_k^{\alpha *}(t_1) x_l^\alpha(t_2)\rangle = \Delta^\alpha\,(t_1, t_2)_{kl}\ . \tag{3.6}$$

The fluctuation x_1 associated to the background R_1 mixes with the field a_0 (the fluctuation of the gauge potential).

We focus on the vertex contained in the term of the effective action (3.2) of type

$$-2\ \mathrm{Tr}\,\Big([R_l, x_j][x_l, x_j]\Big)\,, \tag{3.7}$$

which gives a vertex with two massive bosons and a massless one and another one with all three massive bosons. Focusing on the second case and choosing a particular combination of the roots we obtain a term of the type

$$\mathbf{R}^{\alpha^1}\cdot\mathbf{x}^{\alpha^2}\,\mathbf{x}^{\alpha^1}\cdot\mathbf{x}^{\alpha^3} \equiv v^{23}\,t\,x_1^{13}x_j^{23}x_j^{12} + b_l^{23}x_l^{13}x_j^{23}x_j^{12}\,, \tag{3.8}$$

The two-loop setting-sun diagram in Fig. 1, which is obtained from the insertion of

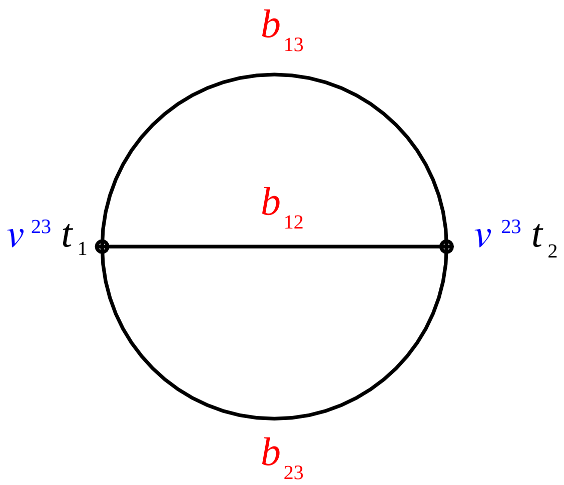

Figure 1: The setting-sun diagram.

two of the vertices (3.8), yields a contribution given, up to overall numerical factors, by

$$\begin{aligned} a_\ominus = \int \mathrm{d}T \int \mathrm{d}t\, &\langle \mathbf{R}^{\alpha^1}(t_1)\cdot\mathbf{x}^{\alpha^2 *}(t_1)\,\mathbf{R}^{\alpha^1}(t_2)\cdot\mathbf{x}^{\alpha^2}(t_2)\rangle\,\times \\ &\times\langle \mathbf{x}^{\alpha^1 *}(t_1)\cdot\mathbf{x}^{\alpha^3 *}(t_1)\mathbf{x}^{\alpha^1}(t_2)\cdot\mathbf{x}^{\alpha^3}(t_2)\rangle\,. \end{aligned} \tag{3.9}$$

In eq. (3.9), we defined $t = (t_1 - t_2)/2$, $T = (t_1 + t_2)/2$.

Our strategy is to do first the integration over the time difference t. Because distances are large, this leaves us with a local expression $\mathcal{L}$ in the $\mathbf{R}^\alpha(T)$ and their derivatives.

The propagator (3.6) can be written as

$$(\Delta^\alpha)_{kl} = \frac{1}{R^\alpha}\int_0^{(R^\alpha)^2/\dot{R}^\alpha}\frac{\mathrm{d}\,u\,e^{-u}}{\sqrt{4\pi u}}\,W_{kl}(u;\,\dot{R}^\alpha,\,R^\alpha)\,e^{-\frac{(R^\alpha)^2}{u}t^2}\,, \tag{3.10}$$

where W_{kl} is defined in appendix B.

We note first that the integration over u can be extended to infinity, up to exponentially negligible corrections for $(R^\alpha)^2/\dot{R}^\alpha \to \infty$.

The important point is that for the heavy propagators Δ^{α^2} and Δ^{α^3}, we can put $W_{kl} = \delta_{kl}$, up to terms of order not less than $(\dot{R}^{\alpha^{2,3}})^2/(R^{\alpha^{2,3}})^4$ which are not relevant for our computation, as it will become clear in the following. Thus, to the order in which we are interested, we can replace the heavy propagators with those obtained as if R^{α^2} and R^{α^3} were constant, that is

$$(\Delta^{\alpha^2})_{kl} = \delta_{kl}\,\frac{1}{2\,R^{\alpha^2}}e^{-2\,R^{\alpha^2}|t|}\,, \tag{3.11}$$

and similarly for Δ^{α^3}. Further, with the same accuracy, we can let $R^{\alpha^1}(t_1) \sim R^{\alpha^1}(t_2) \sim R^{\alpha^1}(T)$, since $t \leq 1/(R^{\alpha^2} + R^{\alpha^3})$, and, for the same reason we can put $\Delta^{\alpha^1}(T,t) \sim \Delta^{\alpha^1}(T, t = 0)$.

We can now perform the integration over t, obtaining

$$\mathcal{L} \sim \frac{(R^{\alpha^1})^2}{R^{\alpha^2}R^{\alpha^3}}\frac{1}{R^{\alpha^2}+R^{\alpha^3}}\langle \mathbf{x}^{\alpha^1 *}(T)\cdot\mathbf{x}^{\alpha^1}(T)\rangle\,. \tag{3.12}$$

where R^α still depend on T.

A comment is in order. The effective lagrangian (3.12) looks like a one-loop correction to the velocity independent local effective potential:

$$\delta V_{1-loop} = F\,[R^\alpha(T)]\;\mathbf{x}^{\alpha^1 *}(T)\cdot\mathbf{x}^{\alpha^1}(T)\,. \tag{3.13}$$

One could ask whether the potential (3.13) should vanish by supersymmetry reasons, once one performs the sum over all the one-loop diagrams made of heavy fields (including the gauge, ghost and fermion fields). The answer is: no, it is not canceled. In order to verify that this is not the case, we have evaluated the velocity independent part of the sum of all the diagrams with $x^{\alpha^1 *}x^{\alpha^1}$ external lines, for a generic constant background, and found that the term of eq. (3.12), coming from the heavy loop of our setting-sun diagram, does not cancel. In fact, the other heavy loop diagrams carry a different dependence on R^α. This computation is easily done in configuration space, because of the simple expression of the velocity independent propagators, see eq. (3.11).

As a further check, we have also evaluated the velocity-independent effective potential at two loops as a function of the background—that is after performing the integration over the heavy and light fields as well—and found it to be zero, as expected by supersymmetry. Evidently, the cancellations due to the symmetry of the theory do not occur at the level of the first step of integrating only over the heavy fields.

Coming back to our computation, the last propagator Δ^{α^1} in (3.12) can be evaluated by expanding W in powers of u in eq. (3.10). The relevant terms are those of the kind (recall the overall factor $1/R^{\alpha^1}$ in front of (3.10)):

$$\frac{(\dot{R}^{\alpha^1})^6}{(R^{\alpha^1})^{13}}\,u^6\,. \tag{3.14}$$

Performing the last integral over u and substituting into (3.12) we obtain

$$\mathcal{A} \equiv \int \mathrm{d}T\,\mathcal{L} = \int \mathrm{d}T\,\frac{1}{R^{\alpha^2}R^{\alpha^3}(R^{\alpha^2}+R^{\alpha^3})}\,\frac{(\dot{R}^{\alpha^1})^6}{(R^{\alpha^1})^{11}}\,. \tag{3.15}$$

Notice that $\mathcal{A}$ is proportional to the sixth power of $\dot{R}$ and therefore is the first term in the expansion in velocities for which no obvious supersymmetry-induced cancellation is expected [12, 13].

To verify that the action (3.15) contains (2.9) one selects terms proportional to $(\dot{R}^{\alpha^2}\dot{R}^{\alpha^3})^2 = v_{13}^2 v_{12}^2$, and performs the integration over T to get a term of the form

$$\tilde{\mathcal{A}} \sim \frac{v_{13}^2 v_{12}^2}{B^7} \int \mathrm{d}T\, T^4 \frac{(v_{23})^6}{(v_{23}^2 T^2 + b^2)^{11/2}} \sim \frac{v_{13}^2 v_{12}^2 v_{23}}{B^7 b^6} \,. \tag{3.16}$$

The appropriate powers of N_i can be deduced—following [7]— from the double-line notation in which the setting-sun diagram is of order N^3; this factor must be $N_1 N_2 N_3$ for the diagram to involve all three particles. We thus find the term

$$\tilde{a}_{\ominus} \sim \frac{N_1 N_2 N_3 |v_{23}| v_{12}^2 v_{13}^2}{B^7 b^6} \,, \tag{3.17}$$

which reproduces the behavior of the supergravity result (2.9), that is, $\tilde{a}_{\ominus} \sim \tilde{a}_{\vee}$.

Of course, to verify that matrix theory really matches supergravity, one should also check the numerical coefficient as well as the matching of other terms with different powers of B, b and v_{ij}. That would require considering all the various graphs of the YM theory.

Note added in proofs: A paper appeared in the archives [15] in which the complete two-loop computation in matrix theory is performed and shown to precisely agree with the supergravity result. The preliminary analysis of the present work is thus confirmed, contrary to the claims of [16].

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A. The Y diagram

The Y diagram gives a term in the amplitude of the from

$$\begin{aligned} A_Y = &-2\,\kappa^4\,\varepsilon_1 \cdot \varepsilon_1'\,\varepsilon_2 \cdot \varepsilon_2'\,\varepsilon_3 \cdot \varepsilon_3'\,\frac{1}{\mathbf{q}_1^2\,\mathbf{q}_2^2\,\mathbf{q}_3^2} \times \\ &\times \Big\{ (\mathbf{p}_2 - \mathbf{p}_3)^2 \big[\mathbf{q}_3 \cdot (\mathbf{p}_3 - \mathbf{p}_1) + \mathbf{q}_2 \cdot (\mathbf{p}_1 - \mathbf{p}_2)\big] \\ &\quad + (\mathbf{p}_3 - \mathbf{p}_1)^2 \big[\mathbf{q}_3 \cdot (\mathbf{p}_2 - \mathbf{p}_3) + \mathbf{q}_1 \cdot (\mathbf{p}_1 - \mathbf{p}_2)\big] \\ &\quad + (\mathbf{p}_1 - \mathbf{p}_2)^2 \big[\mathbf{q}_2 \cdot (\mathbf{p}_2 - \mathbf{p}_3) + \mathbf{q}_1 \cdot (\mathbf{p}_3 - \mathbf{p}_1)\big] \Big\}^2 . \end{aligned} \tag{A.1}$$

Notice that $A_Y = 0$ whenever $\mathbf{p}_i = \mathbf{p}_j$, as it is also true for $A_\vee$.

In the limiting regime (3.5), and in our particular kinematic configuration, the term (A.1) above turns out to give a contribution proportional to

$$\tilde{a}_Y = \int \mathrm{d}t \prod_i^3 \mathrm{d}\mathbf{q}_i \, e^{i\mathbf{q}_i\cdot(\hat{\mathbf{n}}_1 v_i t + \mathbf{b}_i)} \delta\Big(\sum_j \mathbf{q}_j\Big) A_Y \;=\; f(v)\,\frac{1}{B^9 b^4}\,, \tag{A.2}$$

where $f(v)$ is a homogeneous function of degree five in the velocities v_{ij}, and it is therefore sub-leading with respect to $\tilde{a}_\vee$ in (2.9).

B. The propagators

The explicit form of the propagator is [6]

$$\Delta^\alpha_{kl} = \int \mathrm{d}s \; e^{-|b^\alpha|^2 s} \sqrt{\frac{v^\alpha}{2\pi \sinh 2\, v^\alpha s}} \exp\left\{-v^\alpha (t^2 \coth v^\alpha s + T^2 \tanh v^\alpha s)\right\} \times$$
$$\times \left(\delta_{kl} + v^\alpha_k v^\alpha_l \, \frac{2\,\sinh^2 v^\alpha s}{(v^\alpha)^2}\right). \tag{B.1}$$

By changing the integration variable to

$$u \equiv \frac{(R^\alpha)^2}{\dot{R}^\alpha} \frac{\sinh v^\alpha s}{\cosh v^\alpha s}\,, \tag{B.2}$$

we can rewrite the propagator (B.1) as in eq.(3.10), with

$$W_{kl} = \frac{1}{\sqrt{1 - \frac{(\dot{R}^\alpha)^2}{(R^\alpha)^4} u^2}} \exp\left[-\frac{(\dot{R}^\alpha)^2 (R^\alpha)^2 - (\dot{\mathbf{R}}^\alpha \cdot \mathbf{R}^\alpha)^2}{(R^\alpha)^6} u^3 \sum_{n=0} \left(\frac{\dot{R}^\alpha}{(R^\alpha)^2}\right)^{2n} \frac{u^{2n}}{3+2n}\right] \times$$
$$\times \left(\delta_{kl} + \frac{\dot{R}^\alpha_k \dot{R}^\alpha_l}{(R^\alpha)^4} \frac{2u^2}{1 - \frac{(\dot{R}^\alpha)^2}{(R^\alpha)^4} u^2}\right). \tag{B.3}$$

The important point about W is that it can be expanded in powers of $(\dot{R}^\alpha)^2/(R^\alpha)^4$.

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